

*Further Researches on the Annual Parallax of 61 Cygni.* By Professor Robert S. Ball, LL.D., F.R.S., Royal Astronomer of Ireland.

In the *Dunsink Observations*, Part III., pp. 16–39 (“Observatory,” No. 17, vol. II., p. 137), I have discussed a series of measures of the difference between the declination of 61 A *Cygni* and a star of the 9–10 magnitude north following. The discussion of these observations gave an annual Parallax of  $+0''.4654 \pm 0''.0497$ . Shortly after this series was completed I commenced a new series, in which I employed the following star of 61 *Cygni*, often known as 61 B *Cygni*, instead of the preceding star which had been used in the previous researches. The same comparison star was employed, and the methods of observation and reduction were precisely the same. It is the object of the present paper to describe these observations, and to set forth the results to which they have led.

The series now under discussion was commenced upon September 18, 1878, and was concluded on October 2, 1879.

On each night of observation it was sought to take four complete measures of the difference of declinations. Each measure was derived from four readings of each of the screws in the direct position and four in the reversed position. The final result adopted for each night was the mean of the four separate measures.

To illustrate the nature of these observations and the method employed in their reduction, I here give a series of measures on a night, November 16, 1878, which I have selected at random from the entire series:—

*Micrometric Measurements of the Difference of Declination of 61 B Cygni and Comparison Star on the Night of November 16, 1878.*

Screw I.	Screw II.	Screw I.	Screw II.	Screw I.	Screw II.	Screw I.	Screw II.
63·698	40·627	55·783	32·139	64·318	40·047	55·599	32·446
63·753	40·598	55·857	32·134	64·163	40·177	55·562	32·506
63·791	40·537	55·902	32·063	64·175	40·237	55·501	32·549
63·823	40·537	55·921	32·074	64·097	40·257	55·483	32·633
55·639	32·356	64·217	40·184	55·906	32·143	63·557	40·863
55·666	32·353	64·215	40·142	55·697	32·293	63·293	41·103
55·732	32·268	64·202	40·113	55·716	32·303	63·267	41·088
55·753	32·262	64·213	40·095	55·708	32·348	63·154	41·228

Each of these eight columns has first to be reduced to seconds. This is accomplished by adding together the four larger numbers in each column, and subtracting from the result the sum of the four smaller numbers: *e.g.*, for the first column this result is 32·275. This has to be multiplied by one-eighth of the value of the micrometer Screw I., at the temperature of

the instrument ( $39^{\circ}9$ ). The result thus found is  $36''292$ . In a similar manner the movement of Screw II. in the second column gives  $37''224$ . Thus the first set of measures on the night in question gives for the apparent total distance the result  $73''516$ . This must, however, receive a correction of  $+0''026$  on account of refraction, and of  $-0''031$  on account of reduction for Aberration, Precession, and Nutation,\* so that the final result is  $73''511$ . In a similar manner we have for each of the other sets of observations on the night in question  $73''705$ ,  $73''541$ ,  $73''453$ . The mean of the whole four is  $73''552$ , which is accordingly taken as the final result of the observations for this one night.

In some cases the number of observations made was less than four, so it has been found convenient when estimating the weight attached to each night's work to employ simply the number which denotes the number of complete measures which have been obtained. Thus, for the night just considered, the weight is taken as 4.

In the following table is given the results of the observations on the several nights which are included in the present series:—

*Mean Results of the Observations of the Difference in Declination between 61 B Cygni and Comparison Star reduced for refraction and reduction.*

Date.	Diff.	Date.	Diff.	Date.	Diff.
1878.		1878.		1879.	
Sept. 18	73'657	Nov. 17	73'609	Apr. 16	72'348
20	73'633	21	73'507	30	72'117
23	73'579	25	73'615	May 4	72'229
25	73'613	28	73'437	8	71'907
Oct. 23	73'520	Dec. 1	73'554	15	71'602
24	73'698	5	73'436	22	71'640
25	73'528	8	73'442	June 10	71'530
28	73'739	30	73'268	15	71'345
29	73'537	1879.		28	71'169
31	73'707	Jan. 8	73'400	July 26	70'853
Nov. 1	73'802	15	73'370	Sept. 17	70'424
8	73'633	18	73'325	Oct. 2	70'816
16	73'552	Apr. 10	72'512		

A glance at these results exhibits in a conspicuous manner the large proper motion of 61 B Cygni relatively to the star which has been chosen for comparison. To clear the observations from the grosser part of the effects of proper motion, it

\* The methods of correcting such observations for refraction and reduction are similar to those already described in *Dunsink Observations*, Part III. pp. 19-22.

became necessary to adopt an approximate value of the relative annual proper motion in declination. The former series of observations had conclusively established the fact that the comparison star is not affected by any considerable proper motion. I therefore felt warranted in assuming Argelander's value of the absolutely proper motion of 61 B *Cyg.* in declination as approximately the relative proper motions. The value in question is  $+3''.016$  (*Positiones Medix*, p. 27).

The mean epoch of the entire series of observations is 1879.069. The mean value of the observed differences of declination is  $72''.85549$ . The assumed epoch is 1879.0, whence applying a correction of  $+0.00065$  for proper motion, we have as the adopted mean difference of declinations at the epoch  $73''.06414$ .

The equations of condition are now formed in the usual manner.

The true mean distance at the epoch is assumed to be  $73''.06414 - x$ ; the true value of the proper motion in declination is assumed to be  $3''.016 + x'$ ; the correction to be applied to an observed difference of declinations for the parallax  $\varpi$  is  $-[9.92229]\cos(\odot - 121^\circ 39')R\varpi$ ; the correction to be applied for a possible difference  $\kappa$  between the coefficients of aberration of the two stars is  $-[9.92229]\sin(\odot - 121^\circ 39')\kappa$ .

61 B *Cygni*.

Equations of Condition.	Weight.	Residual.
$-x + .2868x' + .4907\varpi + .6785\kappa - .272 = 0$	2	$-.042$
$+ .2814 + .4670 + .6947 - .280 = 0$	2	$-.060$
$+ .2732 + .4301 + .7177 - .309 = 0$	2	$-.103$
$+ .2675 + .4041 + .7325 - .258 = 0$	2	$-.062$
$+ .1910 + .0177 + .8360 - .120 = 0$	1	$-.092$
$+ .1882 + .0026 + .8362 + .066 = 0$	4	$+.087$
$+ .1855 - .0111 + .8361 - .096 = 0$	2	$-.080$
$+ .1773 - .0545 + .8344 + .140 = 0$	2	$+.135$
$+ .1745 - .0697 + .8332 - .053 = 0$	2	$-.064$
$+ .1692 - .0973 + .8304 + .133 = 0$	4	$+.110$
$+ .1665 - .1116 + .8286 + .236 = 0$	4	$+.206$
$+ .1473 - .2110 + .8085 + .125 = 0$	2	$+.048$
$+ .1255 - .3198 + .7710 + .109 = 0$	4	$-.020$
$+ .1227 - .3332 + .7652 + .175 = 0$	4	$+.040$
$+ .1122 - .3838 + .7403 + .105 = 0$	2	$-.053$
$+ .1009 - .4355 + .7101 + .247 = 0$	4	$+.063$
$+ .0928 - .4718 + .6858 + .093 = 0$	4	$-.110$
$+ .0843 - .5074 + .6589 + .236 = 0$	4	$+.016$
$+ .0735 - .5511 + .6214 + .150 = 0$	4	$-.092$

Equations of Condition.	Weight.	Residual.
$+0.0652 - 0.5832 + 0.5902 + 0.181 = 0$	4	-0.079
$+0.0054 - 0.7587 + 0.3222 + 0.188 = 0$	4	-0.167
$-0.0200 - 0.7996 + 0.1947 + 0.396 = 0$	2	+0.015
$-0.0391 - 0.8175 + 0.0922 + 0.424 = 0$	4	+0.029
$-0.0473 - 0.8214 + 0.0481 + 0.404 = 0$	2	+0.005
$-0.2730 - 0.1566 - 0.8215 + 0.271 = 0$	1	+0.140
$-0.2920 - 0.0717 - 0.8331 + 0.165 = 0$	1	+0.074
$-0.3274 + 0.1275 - 0.8265 + 0.040 = 0$	2	+0.047
$-0.3383 + 0.1831 - 0.8162 + 0.185 = 0$	4	+0.219
$-0.3491 + 0.2382 - 0.8022 - 0.104 = 0$	1	-0.043
$-0.3685 + 0.3332 - 0.7686 - 0.351 = 0$	2	-0.241
$-0.3876 + 0.4226 - 0.7246 - 0.255 = 0$	2	-0.098
$-0.4394 + 0.6307 - 0.5599 - 0.209 = 0$	2	+0.082
$-0.4530 + 0.6763 - 0.5061 - 0.353 = 0$	1	-0.056
$-0.4887 + 0.7717 - 0.3507 - 0.421 = 0$	2	-0.065
$-0.5657 + 0.8485 + 0.0297 - 0.505 = 0$	4	-0.079
$-0.7100 + 0.5067 + 0.6669 - 0.499 = 0$	2	-0.175
$-0.7516 + 0.3162 + 0.7741 + 0.019 = 0$	2	+0.266

From these I obtain the following normal equations:—

$$\begin{aligned}
 +97.000 x + 3.4413 x' + 10.009 \varpi - 36.235 \kappa - 3.9766 &= 0 \\
 + 3.4413 + 7.5700 - 5.9279 + 9.0188 + 2.7095 &= 0 \\
 + 10.009 - 5.9279 + 22.683 - 11.831 - 10.504 &= 0 \\
 - 36.235 + 9.0188 - 11.831 + 44.890 + 3.9185 &= 0
 \end{aligned}$$

Solving these equations I obtain

$$\begin{aligned}
 x &= +0.0227 \pm 0.0194 \\
 x' &= -0.0878 \pm 0.0705 \\
 \varpi &= +0.4676 \pm 0.0321 \\
 \kappa &= +0.0719 \pm 0.0314
 \end{aligned}$$

These values when substituted in the equations give the residuals in the column following the equations of condition. The sum of the squares of the absolute terms is 6.172, while the sum of the squares of the residuals is but 1.223, the weights being of course taken into account in each case.

The probable error of one observation is  $\pm 0''.1298$ , whence the probable error of the mean of four observations, which form a complete series for one night, is  $\pm 0''.0649$ .